Direct Extraction of FET Noise Models From Noise Figure Measurements

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***Abstract*—An algorithm is presented that allows for noniterative extraction of the parameters of the Pucel and Pospieszalski FET noise models directly from noise-figure measurements. Since the goal is to minimize the number of source–pull measurements, the number of different source admittances required as a minimum to determine the model parameters reliably is investigated. It turns out that, in the case of the Pospieszalski model, 50-Ω measurements are sufficient, while in case of the Pucel model, three additional source impedances have to be taken into account. The results are verified by investigating MESFET and pseudomorphic high electron-mobility transistor devices.**

***Index Terms*—Equivalent circuits, MESFETs, MODFETs, noise measurement, semiconductor device modeling, semiconductor device noise.**

I. Introduction

In order to describe the RF noise of a MESFET or high electron-mobility transistor (HEMT), one has to determine the parameters of the noise model in addition to the small-signal equivalent-circuit elements. Although, in principle, a set of n linearly independent equations is sufficient to determine n unknowns, in practice, a much larger number of measurements is required to extract the two to four unknowns of the noise models. Thereby, measurement inaccuracies cancel out and the accuracy of the model parameters and stability of the extraction algorithm are enhanced.

Most algorithms rely on the source–pull measurement technique to extract the noise parameters from a large set of parameters at a single frequency, e.g., by employing the correlation matrix method [1] to deembed the parasitics and to determine the intrinsic noise sources. The drawback of source–pull measurement is that these measurements are both elaborate and time consuming since they require expensive tuner systems and frequent calibration. A way to bypass this problem and thereby to facilitate and speed up the measurement procedure, is to take the frequency dependence of the noise factors into account. This is possible in the microwave range, where the parameters of the noise model are assumed to be frequency independent.

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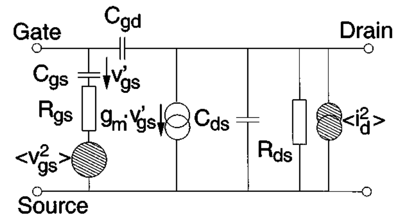

Fig. 1. Intrinsic equivalent circuit of the FET including noise sources.

This approach is applied in several algorithms that have been proposed to calculate the noise model parameters directly from the noise figures (NFs) measured at 50-Ω source impedance [2]–[4]. In these algorithms, the underlying noise models are simplified, or exactly 50-Ω source impedance is required. In contrast, the algorithm presented here (which is based on [5]), allows for direct noniterative extraction of the parameters of the Pospieszalski [6] and Pucel *et al.* [7] noise models without simplifications. Furthermore, the algorithm is not restricted to exactly 50-Ω source impedance, which is difficult to realize in a broad frequency range. The algorithm calculates the noise model parameters directly from a linearly independent set of noise factors, measured varying source impedance or frequency, or both.

The disadvantage of single-source impedance measurements, however, is that even a large number of measurements performed in a broad frequency range may result in a set of weakly linear dependent equations. It is, therefore, the aim of this paper to investigate to which extent the number of source impedances can be reduced without sacrificing accuracy in the extraction.

MESFETs as well as pseudomorphic high electron-mobility transistors (pHEMTs) in the frequency range of 4–26 GHz are considered.

II. Model Parameter Extraction

We consider the two most common noise models for MESFETs and HEMTs, i.e., the Pospieszalski and Pucel *et al.* models. Both models introduce two noise sources in the intrinsic equivalent circuit shown in Fig. 1. In Pospieszalski's model, elevated temperatures T_g and T_d are assigned to the resistances $R_{\rm gs}$ and $R_{\rm ds}$. Both resistances contribute uncorrelated thermal noise. In Pucel *et al.*'s model, the short-circuit noise currents at drain $\langle i_d^2 \rangle$ and gate $\langle i_g^2 \rangle$ are modeled by $\langle i_g^2 \rangle = R4kBT_0(\omega C_{\rm gs})^2/g_m$, $\langle i_d^2 \rangle = P4kBT_0 g_m$, with

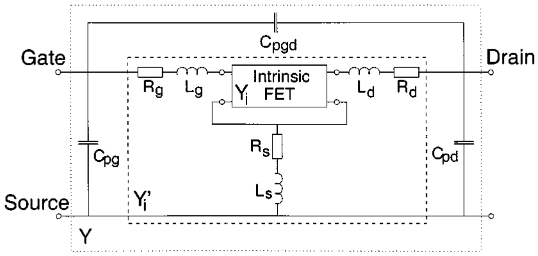

Fig. 2. Extrinsic embedding network. The resistances exhibit thermal noise.

$\langle i_g i_d^* \rangle = C\sqrt{RP}4kBT_0\omega C_{\rm gs}$. The Pospieszalski model, therefore, has two parameters T_g and T_d, while Pucel's model needs the parameters R, P, and C, where C is a complex number. To calculate the noise factor, the correlated short-circuit noise currents of the entire FET (see Fig. 2) $\langle i_{r1}^2 \rangle$ and $\langle i_{r2}^2 \rangle$ are first calculated. The NF of the FET at a given source admittance Y_q is determined from the short-circuit current. Relating the short-circuit output noise current in case of a noisy and noise-free FET, i.e., $\langle i_k^2 \rangle$ and $\langle i_{k0}^2 \rangle$, respectively, one obtains the following for $\mathrm{NF} = \langle i_k^2 \rangle / \langle i_{k0}^2 \rangle$:

$$\mathrm{NF} = 1 + \frac{(\mathbf{AB})\mathbf{C}(\mathbf{AB})^\dagger}{\langle i_{k0}^2 \rangle} + \frac{(\mathbf{AD})\mathbf{E}(\mathbf{AD})^\dagger}{\langle i_{k0}^2 \rangle} \qquad (1)$$

where the $\dagger$ denotes the Hermitian conjugate and $\langle i_{k0}^2 \rangle = |Y_{21}/(Y_q + Y_{11})|^2 \langle i_q^2 \rangle$, where Y denotes the Y-parameters of the entire FET, Y_i' denotes the Y-parameters of the FET without the extrinsic capacitances, and Y_i and Z_i represent the Y- and Z-parameters of the intrinsic FET. Y_q is the admittance of the source, whose thermal noise current is given by $\langle i_q^2 \rangle = 4kT_0 B \mathrm{Re}\{Y_q\}$. T_0 is the ambient temperature, k denotes the Boltzmann constant, and B is the noise bandwidth.

The matrices are given by

$$\mathbf{A} = \left(\left[\frac{Y_{21}}{Y_q + Y_{11}}\right] Y'_{i,11} \quad \left[\frac{Y_{21}}{Y_q + Y_{11}}\right] Y'_{i,12} \;\; -Y'_{i,21} \;\; -Y'_{i,22} \right)$$

$$\mathbf{B} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\mathbf{C} = 2kT_0 B \begin{pmatrix} R_s & 0 & 0 \\ 0 & R_g & 0 \\ 0 & 0 & R_d \end{pmatrix}.$$

The matrix $\mathbf{E}$ contains the noise parameters of the intrinsic FET

$$\mathbf{E} = 2kB \begin{pmatrix} T_g & 0 \\ 0 & T_d \end{pmatrix}$$

for the Pospieszalski model, and in case of the Pucel *et al.* model

$$\mathbf{E} = 2kBT_0 \begin{pmatrix} R & C\sqrt{RP} \\ C^*\sqrt{RP} & P \end{pmatrix}.$$

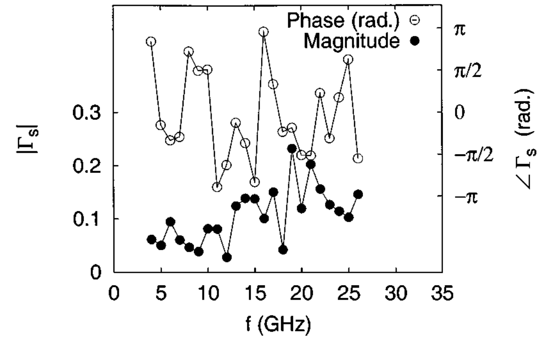

Fig. 3. Source reflection coefficient Γ_s applied in the single source measurements.

Also, the two-column matrix $\mathbf{D}$ is different for the two models

$$\mathbf{D} = \begin{pmatrix} -(\alpha Z_{i,11} + \beta Z_{i,12}) & -\gamma Z_{i,12} \\ -(\alpha Z_{i,21} + \beta Z_{i,22}) & -\gamma Z_{i,22} \\ -(\alpha Z_{i,11} + \beta Z_{i,12}) & -\gamma Z_{i,12} \\ -(\alpha Z_{i,21} + \beta Z_{i,22}) & -\gamma Z_{i,22}\gamma \end{pmatrix}$$

with

$$\alpha = \sqrt{R_{\rm gs}}(Y_{i,11} + Y_{i,12})$$
$$\beta = \sqrt{R_{\rm gs}}(Y_{i,21} - Y_{i,12})$$
$$\gamma = 1/\sqrt{R_{\rm ds}}$$

for the Pospieszalski model, and for Pucel *et al.*'s model

$$\alpha = (\omega C_{\rm gs})/\sqrt{g_m}$$
$$\beta = 0$$
$$\gamma = \sqrt{g_m}.$$

Equation (1) represents a linear equation of the unknown model parameters T_g and T_d or P, R, and $C\sqrt{PR}$, respectively. From several measurements that generate a set of linearly independent equations, these parameters, therefore, can be determined directly by a linear least-squares algorithm. In order to do so, (1) is rearranged so that the unknowns are now contained in a single vector $\mathbf{X}$ instead of the matrix $\mathbf{E}$. In the case of the Pucel *et al.* model, two new elements are formed from the above matrices and vectors as follows:

$$\mathbf{G} = \left(\mathbf{AD}^{(1)} \quad \mathbf{AD}^{(2)} \right) \qquad (2)$$

$$\mathbf{H} = \begin{pmatrix} \mathbf{AD}^{(1)} & 0 \\ 0 & \mathbf{AD}^{(2)} \\ \mathbf{AD}^{(2)} & \mathbf{AD}^{(1)} \\ -j\mathbf{AD}^{(2)} & j\mathbf{AD}^{(1)} \end{pmatrix}.$$

$\mathbf{D}^{(1)}$ and $\mathbf{D}^{(2)}$ denote the first and second columns of matrix $\mathbf{D}$, respectively. The vector of the unknowns is

$$\mathbf{X} = \left(R \;\; P \;\; \mathrm{Re}\{C\}\sqrt{RP} \;\; \mathrm{Im}\{C\}\sqrt{RP} \right)^T$$

For the Pospieszalski model, $\mathbf{G}$ has the same form, as in (2) and

$$\mathbf{H} = \begin{pmatrix} \mathbf{AD}^{(1)} & 0 \\ 0 & \mathbf{AD}^{(2)} \end{pmatrix}$$

$$\mathbf{X} = (T_g \quad T_d)^T.$$

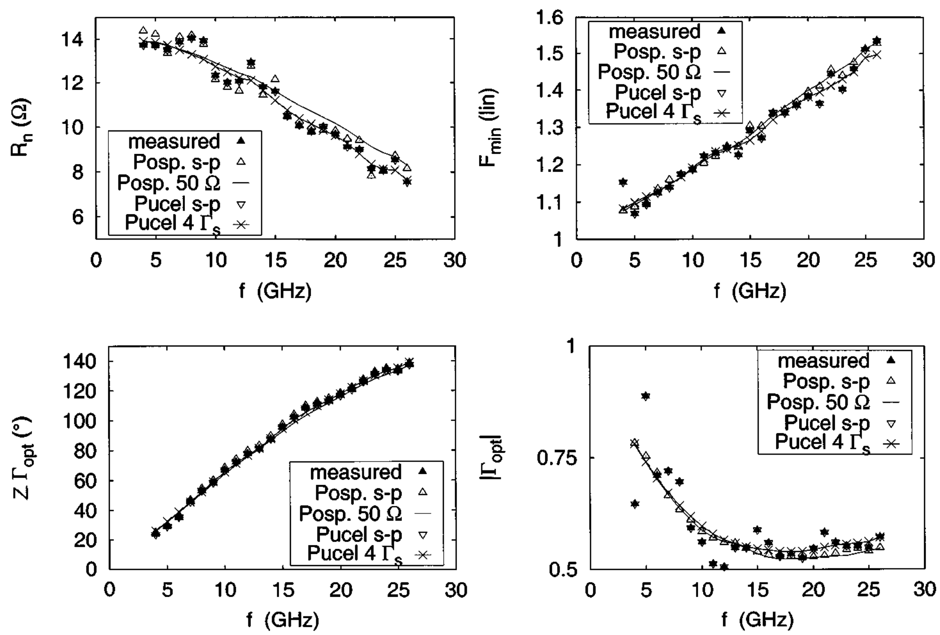

Fig. 4. Noise parameters $F_{\min}$, R_n, $|\Gamma_{\rm opt}|$, and $\angle\Gamma_{\rm opt}$ of the HEMT at $V_{\rm ds} = 3$ V and $I_d = 15$ mA. Measurements (▲) simulated with Pospieszalski (△) and Pucel *et al.* (▽) models extracted at every frequency point from full source–pull data, simulated with the Pospieszalski model from 50-Ω measurements (—), and with the Pucel *et al.* model extracted from a minimum set of source impedances (-×-).

For both models, the equation for each measured noise factor NF is written by

$$\mathrm{NF} = 1 + \frac{(\mathbf{AB})\mathbf{C}(\mathbf{AB})^\dagger}{\langle i_{k0}^2\rangle} + \frac{(\mathbf{GH}^\dagger)}{\langle i_{k0}^2\rangle}\mathbf{X}. \tag{3}$$

Finally, the equation that one has to solve is found to be

$$\mathrm{Min} \overset{!}{=} \sum_{n=1}^{N}\left[1 - \mathrm{NF}_n + \frac{(\mathbf{A}_n\mathbf{B}_n)\mathbf{C}(\mathbf{A}_n\mathbf{B}_n)^\dagger}{\langle i_{k0,n}^2\rangle} + \frac{(\mathbf{G}_n\mathbf{H}_n^\dagger)}{\langle i_{k0,n}^2\rangle}\mathbf{X}\right]^2 \tag{4}$$

with the number of measurements N.

III. Results

MESFET and pHEMT devices were measured on-wafer. Source–pull noise measurements up to 26 GHz with 13 source admittances were performed in order to determine the four two-port noise parameters $F_{\min}$, R_n, and $\Gamma_{\rm opt}$ of the FETs. For details of the measurement setup and FET device parameters, see [8]. The source reflection coefficient that is used in the single source extractions is shown in Fig. 3. These values correspond to the tuner settings closest to 50 Ω ($\Gamma_s = 0$).

First, noise model parameters are determined from the full set of source–pull data for every frequency. For both models, the parameters can be extracted reliably (Fig. 4). In a second step, the minimum number of source admittances is determined. It turns out that the two parameters of the Pospieszalski model can be extracted from a set of NFs measured at various frequencies with a fixed source reflection coefficient (Fig. 5). In order to extract the parameters of the Pucel *et al.* model (Fig. 6), measurements with three additional source impedances have to be performed. Since the location of the source reflection coefficients in the Smith chart determines whether or not the resulting equations are linearly independent, $|\Gamma_s| \approx 0.5$ with $\angle\Gamma_s \approx 0°, 90°, -90°$ is chosen. These values are not too close together and are expected to yield moderate NFs. Measurements performed at Γ_s that lead to high NFs may lead to difficulties when determining the value ($F_{\min}$) and location in the Smith chart ($\Gamma_{\rm opt}$) of the

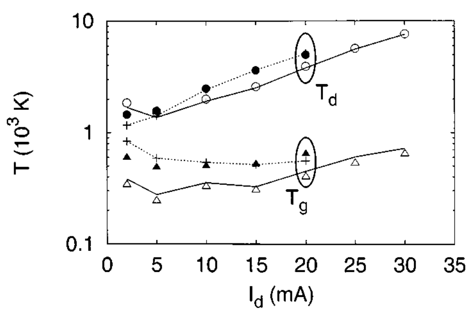

Fig. 5. Noise temperatures T_g and T_d of the Pospieszalski model, mean values extracted from source–pull data (HEMT: empty symbols, MESFET: solid symbols), and from 50-Ω measurement (HEMT: solid lines, MESFET: dashed lines with crosses).

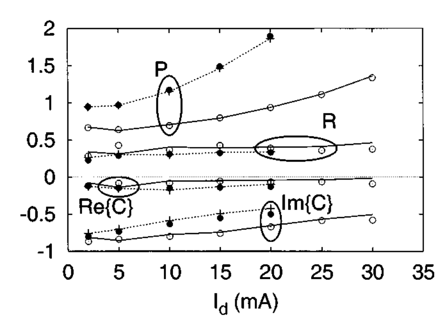

Fig. 6. Noise parameters P, R, and C of Pucel *et al.* model, mean values extracted from full source–pull data (○ HEMT, ● MESFET), and from measurement with four source impedances (HEMT: solid lines, MESFET: dashed lines with crosses).

minimum NF. It turns out that the extraction is reliable for the devices-under-test at all drain currents investigated here. Further reduction of the number of source reflection coefficients leads to instabilities in the extraction algorithm. While the measurements themselves still are modeled well, the extracted parameters cannot be used to predict the noise behavior of the device under different conditions. This indicates that the equations are weakly linear dependent, and do not provide enough information to extract the unique set of parameters.

IV. CONCLUSIONS

An algorithm has been presented that allows for extraction of the parameters of the Pospieszalski and Pucel *et al.* noise models directly from NF measurements. It is not necessary to determine the four two-port noise parameters first. Thereby, measurements at different frequencies as well as common source–pull measurements at different source reflection coefficients can be taken into account. In the microwave region, where the model parameters are frequency independent, measurements at different frequencies can be used to increase the accuracy of the extraction.

It has also been investigated to which extend the source–pull efforts can be reduced without sacrificing accuracy. While only measurements near $\Gamma_s = 0$ are required in case of the Pospieszalski model, four different source impedances are required to determine the parameters of the Pucel *et al.* model. It, therefore, can be concluded that the algorithm presented here allows to significantly reduce measurement efforts when the Pospieszalski model is used. It also can be concluded that the two additional parameters of the Pucel *et al.* model prevent parameter extraction at a fixed source since, with two additional degrees of freedom, the system is no longer well determined.

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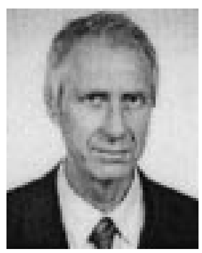

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